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Notch Sensitivity in Steel

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INTRODUCTION

Recent investigations both in Europe and America have shown that the percentage of service failures in machine parts caused by fatigue is as high as 80 to 95 per cent. Consequently, all equipment and machinery liable to be subjected to any kind of vibration, rotation or repeated shock loads must be properly designed to resist fatigue. The fatigue strength of a machine part is influenced by a number of factors. Some of these can be related to the behaviour of the load on the machine part and consequently to the stresses in the part. The stress factors include, for example, the magnitude of steady and alternating stresses and their variations with time, the frequency of the alternating stresses and the periodicity with which these stresses occur. Another group of factors can be associated with the shape and size of a machine part. This group is dominated by the notch effect, which occurs as a result of the shape of the part, as, for example, in fillets, around holes, in grooves. Furthermore, the notch effect occurs in the surface layer because of the profile depth in the surface. The relationship between the fatigue strength and the absolute size of the cross-section is also associated with this group. A third group can be related to the state prevailing around the part. This refers above all to the influence of the temperature and the possible presence of corrosive media. Finally, the nature of the material used for the part and the state of this material will influence the fatigue strength. Such factors as surface treatment for providing protection against corrosion should also be reckoned upon in this case.

The influencing factors described above can naturally be classified in many different ways, and the simplification made here is not quite correct, since the different factors are often related with one another. Thus, for example, the notch effect due to the shaping of the part is not merely dependent on its shape but also on the material used. This means that one and the same shape will have a different notch effect for different materials.

Unfortunately the origin and nature of fatigue are not yet sufficiently well known to

enable the effect of a certain factor to be predicted exactly. This means that despite comprehensive investigations carried out over more than one century it is still necessary to rely entirely on empirical results. In this respect, the greatest interest has been devoted to the notch effect as influenced by the shaping of a part and its surface finish. Several theories for determining the notch effect have been propounded on the basis of comprehensive tests. An account is given in this paper of an attempt to select the most valuable of these theories and to estimate its accuracy.

The material in this paper was presented, in 1957, in a preliminary form as a technical memorandum at Asea, Västerås, Sweden. Since then, some further information has been obtained concerning the notch effect in steel from a number of tests which have been performed at the Asea laboratories in Västerås and Stockholm. These tests have been based on the results from the first investigation, and the test results have been included in this paper.

DEFINITIONS AND SYMBOLS

The following symbols and designations will be used in this paper.

α = theoretical stress-concentration factor = $\dfrac{\text{maximum peak stress in a notch}}{\text{nominal stress in a notch}}$

In English-speaking countries the factor α is usually represented by K_t or K.

β = fatigue-notch factor = $\dfrac{\text{fatigue strength of unnotched specimen at } N \text{ cycles}}{\text{fatigue strength of notched specimen at } N \text{ cycles}}$

In this paper $N = 10^7$ cycles. In English-speaking countries the factor β is usually represented by K_f.

β_{est} = estimated notch factor

β_{exp} = experimentally determined notch factor

η = notch sensitivity

$\varkappa$ = surface-finish fatigue factor =

$= \dfrac{\text{fatigue strength of specimen with rough surface finish}}{\text{fatigue strength of similar specimen, but polished}}$; [64]

λ = dimensional fatigue factor =

$= \dfrac{\text{fatigue strength of specimen of } d \text{ mm diam.}}{\text{fatigue strength of specimen of 10 mm diam.}}$; [47]

ϱ = radius of curvature at the base of the notch, in mm

ϱ^* = material constant, used by Neuber

$\hat{\sigma}$ = best estimate of dispersion in $(\bar{C} - \bar{L})$

σ_B = ultimate strength of material, in kg/mm²

$\hat{\sigma}_C$ = best estimate of dispersion in strength

σ_D = fatigue strength

$\hat{\sigma}_L$ = best estimate of dispersion of load

σ_S = yield point, in kg/mm²

$\hat{\sigma}^2$ = best estimate of variance

χ = stress gradient $= \frac{1}{\sigma} \times \frac{d\sigma}{dx}$, in mm^{-1} [5, 8, etc.]
ω = flank angle
a_1, a_2 = material constants, used in Heywood's theory
C = strength
d = minimum diameter at the root of the notch, in mm
d_1 = diameter of transverse hole, in mm
D = maximum diameter adjacent to the notch, in mm
F = variance ratio
H = Vickers hardness, in kg/mm^2
i_k = internal notch-effect factor
k_1 = material constant
L = load
n = $f(\chi)$, used in the theory of Siebel and Stieler
o_k = surface-condition factor, used in Petersen's theory
p = significance level, probability
t = depth of notch, in mm
X = β_{exp}/β_{est}

THE FATIGUE-NOTCH FACTOR β

The theoretical stress-concentration factor α is a measure of the disturbances in the flow of the lines of forces occurring in a notch. It can be determined mathematically by means of conventional theories of elasticity, in which the material is assumed to be elastic, homogeneous and isotropic. In practice, however, α is often determined by measuring the stresses with strain gauges or by means of photoelastic investigations.

The fatigue-notch factor β denotes the influence of α on the fatigue strength of a part with a notch. The factor β is less than the factor α. As has already been mentioned, the difference is due to, amongst other factors, the quality of the steel, the magnitude of ϱ and the type of stress. The differences may sometimes be considerable; in extreme cases β amounts to only about 1/10 of α. The fatigue-notch factor β cannot be determined exactly, but it is apparent from the above that an estimated value of β must be found, since dimensions based on α would obviously give excessively large dimensions. Naturally, it would prove impossible to determine β by testing all possible combinations of size and material. Instead, a number of theories have been put forward according to which it is possible to calculate β from α and suitable material constants. Several of these theories are briefly described in the following.

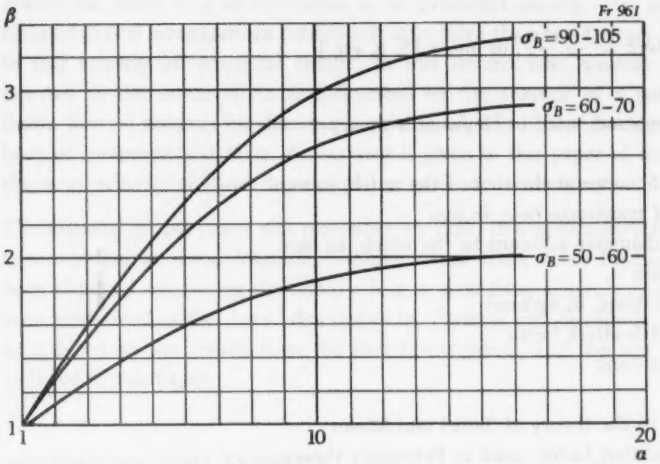


Fig. 1. The fatigue-notch factor, β, as a function of the theoretical stress-concentration factor, α, according to Liljeblad (L).

DIFFERENT THEORIES FOR THE DETERMINATION OF β

Theory of Liljeblad (L) [1]

In this theory the relationship between α and β is obtained from a series of curves based on experiments carried out at Asea. These experiments were started in 1923, and the first results were published in 1926 [1]. The curves are shown in Fig. 1.

Theory of Neuber (N) [2]

In this theory the relationship between α and β is obtained from the equation $\beta = 1 + \eta(\alpha - 1)$, where η is assumed to be a material constant called notch sensitivity. The value of η always lies between zero and unity. In the case of a certain material subjected to a certain stress concentration, η is calculated from the following formula

$$\eta = \frac{1}{1 + \frac{\pi}{\pi - \omega}\sqrt{\frac{\varrho^*}{\varrho}}}$$

Neuber calls ϱ^* "Elementarradius" for the material and gives the value $\varrho^* = 0.48$ mm for carbon steel.

Theory of R. E. Peterson et al. (REP) [3]

According to this theory β is determined in the same manner as in Neuber's theory with the exception that η is a function merely of the radius of the notch. One relationship is obtained for hardened and tempered materials and another one for annealed and normalised materials (see Fig. 2).

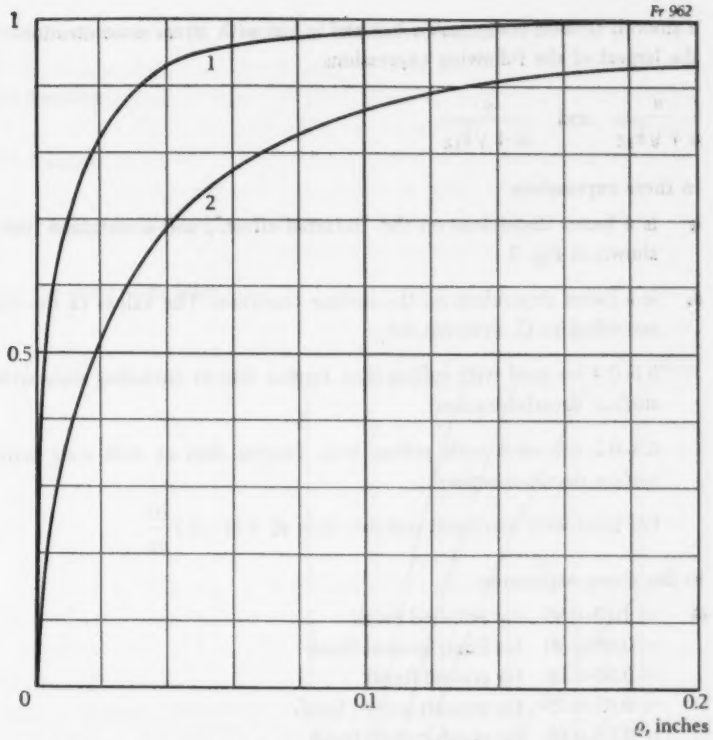


Fig. 2. Notch sensitivity, η, as a function of the curvature at the base of the notch, ϱ, according to R. E. Peterson (REP).

1 Hardened and tempered steel
2 Annealed or normalised steel
The curves are valid for $t/\varrho < 3$, where t = notch depth.

Theory of Bollenrath and Troost (B and T) [4]

From their experimental results Bollenrath and Troost have set forth the following formula

$$\beta = a\left[1 - \frac{\frac{154}{\sigma_B}}{\frac{10}{1+\frac{\sigma_B}{137}} + \varrho}\right]$$

Theory of C. Petersen (CP) [5, 6, 7]

According to this theory β always denotes the ratio between the fatigue strengths for

a smooth tension-compression bar and a bar with stress concentrations. This factor is the largest of the following expressions

$$\frac{a}{i_k+\sqrt{k_1\chi}} \quad \text{and} \quad \frac{a}{o_k+\sqrt{k_1\chi}}$$

In these expressions

i_k is a factor dependent on the "internal effect", and is obtained from the diagram shown in Fig. 3.

o_k is a factor dependent on the surface condition. The values of the factor o_k according to C. Petersen are:

0.6–0.4 for steel with rolling skin, forging skin or annealing scale with insignificant surface decarburisation.

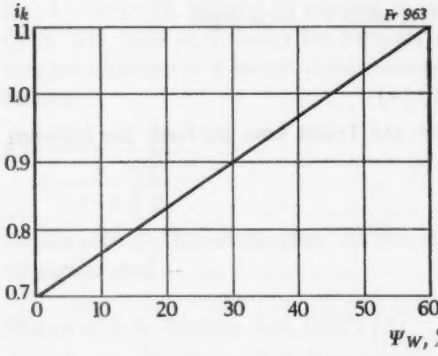
0.3–0.2 for steel with rolling skin, forging skin or mill scale with pronounced surface decarburisation.

For steel with machined surface, $o_k = o_k' + (1-o_k')\frac{20}{\sigma_B}$

In the above expression

o_k' = 1.10–0.95 for polished finish
= 0.95–0.90 for finely ground finish
= 0.90–0.80 for ground finish
= 0.85–0.75 for smooth-turned finish
= 0.75–0.65 for rough-turned finish

k_1 is a material constant which can be calculated with sufficient accuracy from the formula $k_1 = 1600/H^2$, where H is hardness, in kg/mm², according to Vickers.



The factor i_k is obtained in the diagram as a function of the true contraction of the material, Ψ_W, which has been determined from the following formulae:

$$\Psi_W = \frac{\Psi - \Psi_{gl}}{1-\Psi_{gl}} (\Psi = \text{measured contraction})$$

and $\Psi_{gl} \approx 0.3 - \frac{\sigma_B}{500}$

Fig. 3. The internal notch-effect factor, i_k, according to C. Petersen (CP).

The stress gradient χ is defined by the following expressions:

$\frac{6}{d_1}$ for hollow shaft with transverse hole (d_1 = diameter of hole, in mm)

$\frac{2}{\varrho}$ for tension-compression (ϱ = radius of notch, in mm)

$\frac{2}{\varrho}+\frac{2}{d}$ for bending

$\frac{2}{\varrho}+\frac{1}{d}$ for torsion

$\frac{3}{d_1}$ for shaft with transverse hole

$\frac{2}{\varrho}+\frac{4}{D+d}$ for shaft with fillet

Range	Material	Curve	σ_B or $\sigma_{0.2}$ kg/mm²
I	Cast iron	1	$\sigma_B = 15$
		2	$\sigma_B = 30$
II	Cast steel	2	$\sigma_B = 30$
		3	$\sigma_B = 60$
III	Austenitic steel	3	$\sigma_{0.2} = 20$
		4	$\sigma_{0.2} = 40$
IV	Mild steel	4	$\sigma_{0.2} = 15$
		5	$\sigma_{0.2} = 20$
		6	$\sigma_{0.2} = 40$
V	Hardened steel	6	$\sigma_{0.2} = 40$
		7	$\sigma_{0.2} = 50$
		8	$\sigma_{0.2} = 90$
VI	Spring steel	8	$\sigma_{0.2} = 90$
		abscissa	$\sigma_{0.2} > 90$

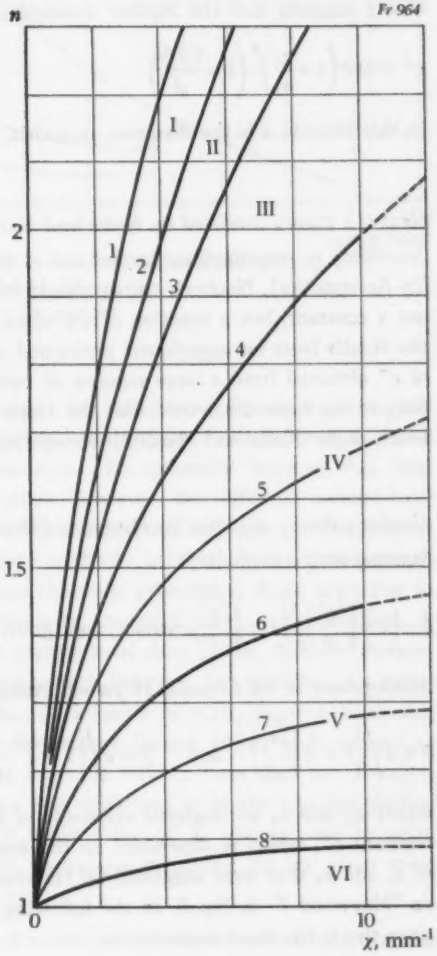


Fig. 4. "Stützfaktor", n, as function of the stress gradient, χ, according to Siebel and Stieler (S and S).

Theory of Siebel and Stieler (S and S) [8]

As is the case with C. Petersen, β in this theory denotes the ratio between the fatigue strength for a smooth tension-compression bar and the fatigue strength for a bar with stress concentrations. The authors use the relationship $\alpha = n\beta$, where n for a certain material is a function of the stress gradient χ. The factor n has been determined for different materials and stress gradients, and the curves in Fig. 4 give mean values of the results. σ_B denotes the ultimate strength, in kg/mm², and $\sigma_{0.2}$ denotes the yield point for 0.2 per cent permanent strain, given in kg/mm².

Neuber's theory modified by Moore (M) [11]

Moore suggests that the Neuber constant ϱ^* should be calculated from the formula

$$\varrho^* = 0.2\left(1 - \frac{\sigma_S}{\sigma_B}\right)^3\left(1 - \frac{0.05}{d}\right)$$

In this formula d is the diameter, in inches, of the specimen at the root of the notch.

Neuber's theory modified by Kuhn and Hardrath (N and K) [10]

According to investigations carried out at the NACA (National Advisory Committee for Aeronautics), Neuber's theory should be modified in the following manner: ϱ^* is not a constant, but a function of the ultimate strength. The diagram in Fig. 5 gives the results from an experiment performed to determine this relationship from values of ϱ^* obtained from a large number of tests on specimens of varying form. In addition, it has been discovered that the factor $\pi/(\pi - \omega)$ exaggerates the effect of the notch angle. Kuhn and Hardrath recommend that this factor should be deleted.

Neuber's theory modified by Heywood (N and H) [12]

Starting with a formula

$\beta = \left(\alpha + \sqrt{\frac{\varrho^*}{\varrho}}\right) \Big/ \left(1 + \sqrt{\frac{\varrho^*}{\varrho}}\right)$, which was given by Neuber in a preliminary stage of development of his formula, Heywood evaluated two new formulae, namely

$$\beta = \alpha \Big/ \left(1 + 2\sqrt{\frac{a_1}{\varrho}}\right) \quad \text{and} \quad \beta = \alpha \Big/ \left(1 + 2\frac{\alpha - 1}{\alpha}\sqrt{\frac{a_2}{\varrho}}\right),$$

where a_1 and a_2 are material constants of the same kind as Kuhn's and Hardrath's constant ϱ^*, which is dependent on the tensile strength of the material. The values of a_1 and a_2 that were suggested by Heywood are given by the curves "Heywood 1" an "Heywood 2" in Fig. 5. In the following, only the latter expression for β is used, since this is the more accurate one.

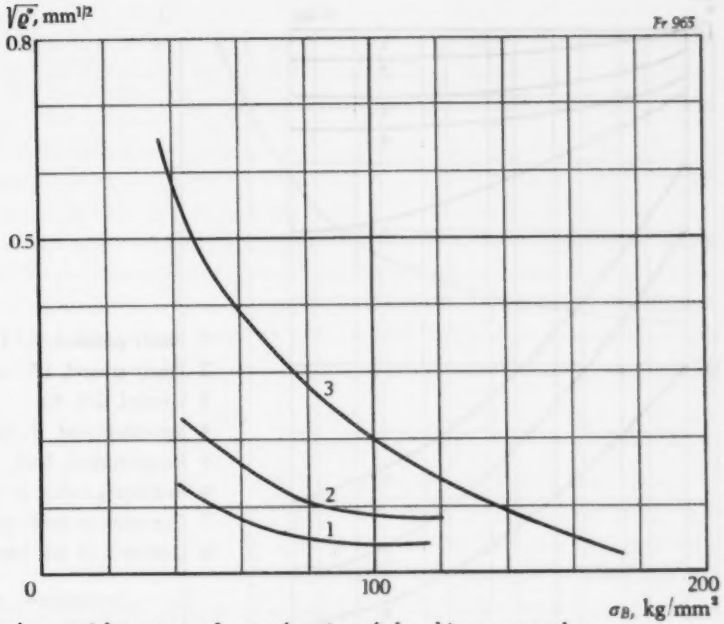
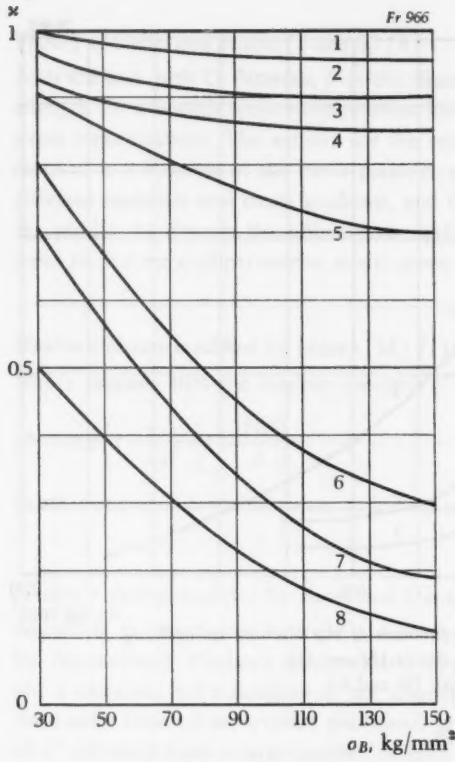


Fig. 5. Neuber's material constant, ϱ^*, as a function of the ultimate strength, σ_B.
1, 2 Neuber–Heywood theory (N and H)
3 Neuber–Kuhn theory (N and K)

A COMPARISON OF THE THEORIES

Several of the theories described in this paper for determining β have been checked by their respective authors, who have compared the estimated notch factor β_{est} and an experimentally determined notch factor β_{exp}. The agreement between β_{exp} and β_{est} was good in several cases. On the whole, however, the different authors have not applied the same experimental data in their comparisons, and thus the theories can hardly be considered as having been collated with one another. If β_{est} is determined for one and the same material with one and the same geometrical shape according to the different theories, considerable variations in the value of β_{est} are obtained. It was therefore decided to apply the same experimental data to the different theories in order to arrive at an unequivocal comparison between the theories. An exact theory should consequently give $\beta_{exp} = \beta_{est}$, or a factor $X = \beta_{exp}/\beta_{est} = 1$. Since β_{exp} is obtained by dividing two empirically determined fatigue limits, each subject to a slight dispersion, and since β_{est} is not exact, it follows then that the X-values will also have a dispersion. The best theory was considered to be the one that yielded the least dispersion in X and where the mean value of X was as near to unity as possible. It would be possible, however, to make a correction for the mean value of X by selecting a suitable correction factor.

A summary was made of a large number of results published in the literature covering



1 Finely polished, 0 – 1μ
2 Finely ground, 1.5 – 2μ
3 Ground, 2.5 – 6μ
4 Smooth-turned, 6 – 16μ
5 Rough-turned, 100μ
6 Remaining rolling or forging skin
7 Corrosion in fresh water
8 Corrosion in salt water

Fig. 6. Diagram of the surface-finish fatigue factor, $\varkappa$, according to Lehr.

β_{exp} in order to obtain a basis for the comparison. In all, more than 1000 β_{exp}-values were obtained. These have been determined for a life of $10^6 < N < 10^7$ load cycles to fracture. As a rule, the fatigue strengths have been determined according to the Wöhler method. The number of specimens included in the Wöhler curves has varied considerably, namely from 2 or 3 bars up to about 30 bars per curve.

The formulae for β_{est} also include α. Since α in many cases is an empirical value, the X-value will also be dependent on the accuracy with which α has been determined. Throughout the investigation one and the same value of α has been used for each type of bar, whereas β_{est} has been calculated according to the different theories. As a rule, α has been determined according to curves and diagrams based on results from accurate measurements of α made at the MPA (Materialprüfungsanstalt), Darmstadt [13, 78]. In some cases, α-values according to Neuber [2] or R. E. Peterson [3] were used.

In the published experimental results the smooth test specimens and those with stress concentrations have occasionally had differing profile depths. In order to prevent this factor from increasing the dispersion in X, the experimentally determined fatigue limits have been recalculated so as to apply to a polished surface. In doing this $\varkappa$ according to Lehr has been adopted (see the diagram in Fig. 6).

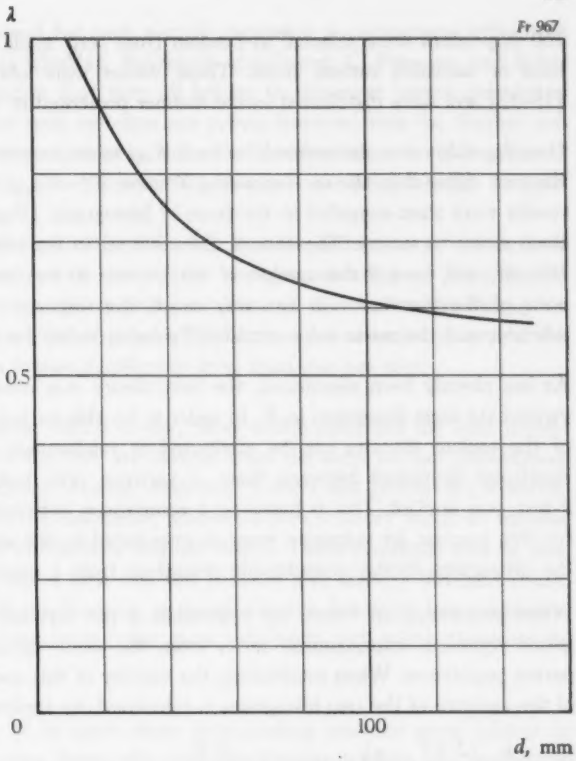


Fig. 7. The dimensional fatigue factor, λ, during bending or torsion as a function of the absolute size according to Lehr.

In some cases, the size of the smooth bars and bars with stress concentrations has differed, and this has been corrected by means of the dimensional fatigue factor, λ, in Fig. 7.

When the fatigue-limit values for smooth bars which have been subjected to tension-compression were not available in the literature, these values have been taken from other sources [13, 15–17].

A complete comparison between the nine theories described above covering all the 1000 β_{exp}-values would imply, amongst other things, that more than 9000 β_{est}-values would have to be determined. A preliminary comparison was made first of all so as to try to lessen the computing work. This was done in such a way that about

TABLE 1

Stress concentration	Type of stress		
	Tension	Bending	Torsion
Groove	16	59	7
Fillet	0	20	10
Transverse hole	13	27	5

150 β_{exp}-values were selected at random from tests made on commonly used normalised or annealed carbon steels. These values were obtained from the references [18–32] and were distributed in the manner presented in Table 1.

One β_{est}-value was determined for each β_{exp}-value according to each of the different theories. After this, the corresponding X-value ($X = \beta_{exp}/\beta_{est}$) was determined. The results were then compiled in the form of histograms (Figs. 10–18) in order to make them easier to survey. The sum of the columns in the histograms is always equal to 100 per cent, even if the number of bars varies. As can be seen from the histograms, none of the theories is in any way exact, the dispersion in some cases being considerable and the mean value occasionally being rather far from unity.

As has already been mentioned, the best theory was considered to be the one that yielded the least dispersion in X. In order to be able to judge whether the divergences of the various theories can be attributed to randomness or whether there are any significant deviations between them, a variance ratio test, often referred to as the *F*-test, was applied. This is based on a comparison between normal distributions, and for this purpose an unknown error is introduced in the comparison, since certain of the histograms display considerable deviations from a normal distribution.

When performing an *F*-test, the hypothesis is put forward that two compared histograms represent experimental series from the same unknown normally distributed parent population. When establishing the validity of this assumption, the best estimate of the variance of the two histograms is calculated, by applying the following expression

$$\hat{\sigma}^2 = \frac{n}{n-1}\left(\frac{\Sigma X^2}{n} - \bar{X}^2\right), \text{ where } \bar{X} = \frac{\Sigma X}{n}$$

The ratio between the larger and the smaller variance of the two histograms is calculated and termed the variance ratio or *F*. Tables have been compiled showing the value of *F* which will be exceeded with a given degree of probability for various sample sizes. If the calculated value of *F* is greater than that given in a particular table, then the probability of the two variances being derived from the same unknown parent population would be less than the significance level *p* of such a table. If this probability, *p*, is very small, the hypothesis can be declared invalid, and it can be asserted that the variances of the two compared histograms differ significantly.

The means ($\bar{X}$) and the variances of the nine histograms have been calculated and are shown in Table 2 together with the number of β_{exp}-values in each histogram.

Of all the histograms which have been compared that of Fig. 18 (Neuber–Heywood) possesses the least variance and has been tested against each of the other histograms. The result of the test can be seen in Table 2. The levels of significance of the corresponding variance ratios are denoted in the Table.

When studying the results from Table 2, the following conclusions can be drawn. The

theory of Neuber and Heywood has such a small dispersion in comparison with those obtained with the theories of Liljeblad, Bollenrath and Troost, C. Petersen, and Siebel and Stieler that the assumption that they all belong to the same parent population can be rejected. The variance ratio test does not prove, however, that the Neuber and Heywood theory is in any way superior to the other theories, since the differences in the dispersion may be due to other factors that have not yet been brought to light.

It has been assumed nevertheless in the following that the difference in the variance between the different histograms is due to the diverging accuracy of the different theories, and that therefore the theory giving evidence of the best results during the test is selected. For this reason, therefore, the "REP"-theory was excluded, even if the value for F does not have a higher significance level than one per cent.

The remaining four histograms, that is to say, those which exhibited the least dispersion and which, according to the test, are derived from the same mother distribution, are all based on Neuber's theory. Among these four, Kuhn's and Heywood's modifications result in a reduction in the dispersion, whereas Moore's theory yields an increase in the dispersion in relation to Neuber's original theory. These variations may be due, however, purely to chance and a new test can possibly give quite a different result. Neuber's theory and Kuhn's modification of it will involve, however, less calculation work than the other theories having the same reliability and might therefore be preferred.

If any of these theories is to be used when dimensioning machine parts subject to fatigue, the dispersion occurring during the calculation work owing to the uncertainty of the theory must be taken into consideration with a suitable factor of safety. The test described up to now was performed on about 150 β_{exp}-values. As has already been mentioned, 1000 β_{exp}-values in all were found in the literature. Since a statistical investigation tends to give more reliable results the greater the population on which it is carried out, it was considered desirable to extend the comparison between the theories to the entire population, but at the same time to limit it to the three best theories "N", "N and K" and "N and H".

When making the comparison, it was thought desirable to divide up the material into two groups, namely ordinary carbon steels in a normalised or annealed state, and alloyed and hardened steels. The former category contained 481 bars distributed as shown in Table 3. β_{est} and X were calculated with this population for the three theories.

The population of alloyed and hardened steels comprised in all 532 bars, distributed as shown in Table 4. β_{est} and X were calculated with this population only for the "N and K", and the "N and H", theories, since the "N"-theory applies merely to carbon steels. (Neuber's constant ϱ^* is given only for ordinary carbon steels.)

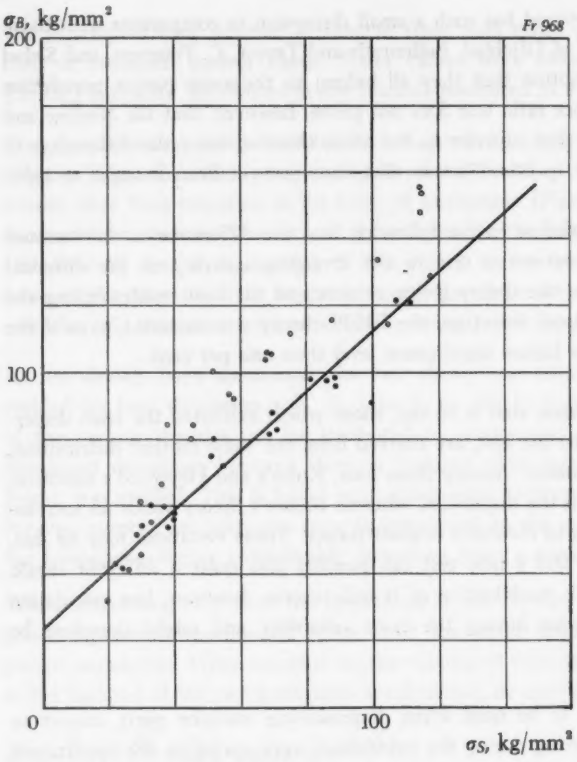


Fig. 8. The ultimate strength, σ_B, as a function of the yield point, σ_S.

Before comparing these populations, however, a modification of the Neuber–Kuhn theory will be described briefly.

MODIFICATION OF THE NEUBER–KUHN THEORY

As has already been mentioned, Kuhn's modification of Neuber's original theory implies that the material constant ϱ^* is replaced by ϱ^* as a function of the ultimate strength (σ_B) of the material. Later on, Heywood modified Neuber's formula, giving a relationship, $\varrho^* = f(\sigma_B)$, similar to that proposed by Kuhn.

The comparison hitherto made between these theories and other theories appears to confirm the validity of this modification, even though the improvement as such is very small. It has been assumed in several papers that a fatigue fracture occurs as a consequence of small plastic deformations in the surface layer of the material under strain, that is to say, something which should be more closely related to the behaviour of the material at the yield point (σ_S) than at the ultimate strength (σ_B). The following experiment was made on the basis of this assumption to attempt to relate the material constant ϱ^* with the yield point (σ_S).

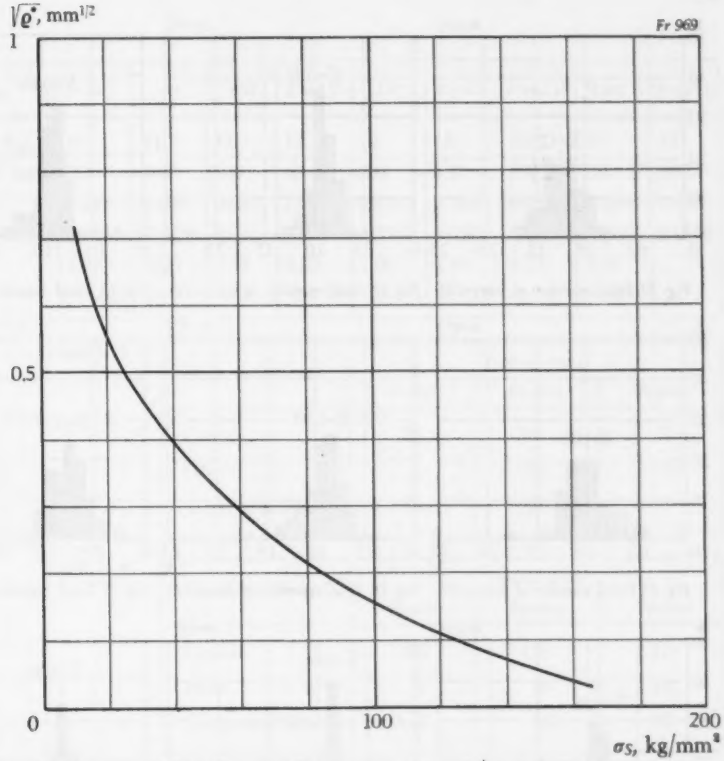


Fig. 9. Relationship between Neuber's material constant, ϱ^*, and the yield point, σ_S, for bars with $0.9 < X < 1.0$.

From the two populations of X-values obtained from Tables 3 and 4 resulting from the application of the Neuber–Kuhn and the Neuber–Heywood theories (in all about 1000 values) a selection was made, on the one hand, of a number of X-values within the interval $0.9 < X < 1.0$, i. e., having good agreement between β_{exp} and β_{est}, and on the other hand, of a number of X-values, where $X < 0.5$, i. e., with bad agreement between β_{exp} and β_{est}. Each of these X-values is naturally associated with a certain combination of σ_B, and σ_S, values, which are characteristic for the corresponding material. These σ_B, and σ_S, values have been plotted in a diagram with σ_S as the abscissa and σ_B as the ordinate (Fig. 8, valid for the Neuber–Kuhn theory). The group of good X-values is designated with dots and the group of inferior X-values with circles. As can be seen, there appears to be a slight difference in the relationship between the yield point and the ultimate strength for the two categories. A similar family of points was obtained for the Neuber–Heywood theory, but this is not shown here. The following modification of the Neuber–Kuhn and the Neuber-Heywood theories was made to elucidate whether this difference had any significance.

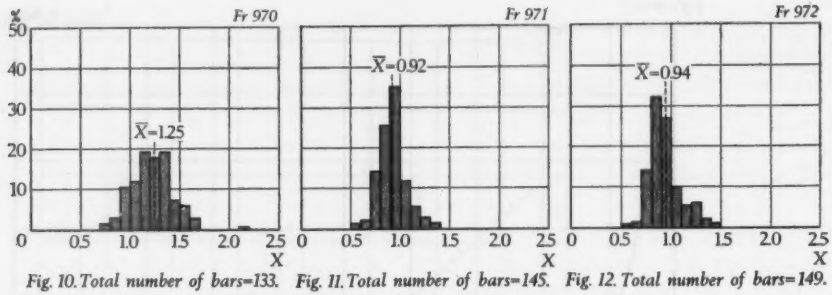


Fig. 10. Total number of bars=133. *Fig. 11. Total number of bars=145.* *Fig. 12. Total number of bars=149.*

Fig. 13. Total number of bars=59. *Fig. 14. Total number of bars=132.* *Fig. 15. Total number of bars=132.*

Fig. 16. Total number of bars=150. *Fig. 17. Total number of bars=152.* *Fig. 18. Total number of bars=150.*

Figs. 10–18. Dispersion in X according to different theories.

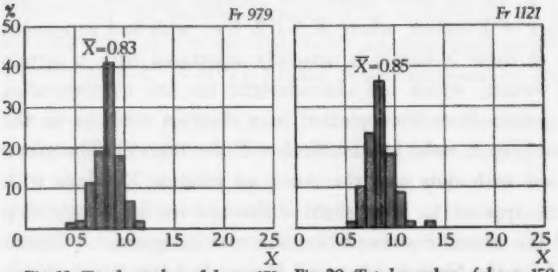


Fig. 19. Total number of bars=153. *Fig. 20. Total number of bars=150*

Figs. 19 and 20. Dispersion in X according to different theories.

Fig. 10. Liljeblad (L)
Fig. 11. Neuber (N)
Fig. 12. R. E. Peterson (REP)
Fig. 13. Bollenrath and Troost (B and T)
Fig. 14. C. Petersen (CP)
Fig. 15. Siebel and Stieler (S and S)
Fig. 16. Neuber–Moore (M)
Fig. 17. Neuber–Kuhn (N and K)
Fig. 18. Neuber–Heywood (N and H)
Fig. 19. Modified Neuber–Kuhn (NKS)
Fig. 20. Modified Neuber–Heywood (NHS)

$\varkappa$ according to Lehr, in Fig. 14 according to C. Petersen.

TABLE 2

Theory	L	N	REP	B and T	CP	S and S	N and M	N and K	N and H
Histogram Fig.	10	11	12	13	14	15	16	17	18
Number of bars	133	145	149	59	132	132	150	152	150
Mean, $\overline{X}$	1.245	0.920	0.937	1.004	0.939	0.765	0.851	0.836	0.855
Variance, $\hat{\sigma}^2$	0.0354	0.0196	0.0261	0.0557	0.0352	0.0334	0.0220	0.0183	0.0180
F	1.97	1.09	1.45	3.20	1.96	1.86	1.23	1.02	*
p	< 0.05	≈ 30	≈ 1	< 0.05	< 0.05	< 0.05	≈ 10	≈ 40	*

TABLE 3

Stress concentration	Type of stress		
	Tension	Bending	Torsion
Groove	29	248	9
Fillet	1	63	17
Transverse hole	58	51	5

TABLE 4

Stress concentration	Type of stress		
	Tension	Bending	Torsion
Groove	106	193	12
Fillet	7	64	24
Transverse hole	144	56	39

A regression line was drawn in the diagram, Fig. 8, through the points from the interval $0.9 < X < 1.0$, and, on the basis of this regression line $\varrho^* = f(\sigma_B)$ was recalculated to $\varrho^* = f(\sigma_S)$, which is illustrated in the diagram in Fig. 9. The values obtained for the bars in Tables 1, 3 and 4 were then recalculated according to this modification of the Neuber–Kuhn theory, called "NKS" and to a similar modification of the Neuber–Heywood theory, called "NHS", where "S" denotes the yield point. The results from the first table are given in Figs. 19 and 20.

A NEW COMPARISON OF THE THEORIES

The values of X and $\hat{\sigma}^2$ were determined for the histograms in Figs. 19 and 20 in the usual manner. It was found that the variance for the "NKS"-theory is less than that for any of the other theories, the "NHS"-theory included. Since it is already apparent from Table 2 that the theory of "N and H" is significantly better than that of "L", or "REP" or "B and T", or "CP", or "S and S", the "NKS", and the "NHS", theories must also naturally be significantly better than these. It was therefore considered unnecessary to compare the "NKS", and the "NHS", theories with these, but only with the theories of "N", "N and M", "N and K", and "N and H". The results of this comparison are shown in Table 5.

From this Table it is evident at once that the "REP"-theory can be disregarded in comparison with the "NKS"-theory, since $p = 0.1$ per cent. Further, it may be possible to disregard the "N and M"-theory in comparison with the "NKS"-theory (p being 2.5 per cent).

The "NHS"-theory yields a very small reduction in $\hat{\sigma}^2$ in comparison with the "N and H"-theory ($\hat{\sigma}^2 = 0.0173$ instead of 0.0180). In addition, since the "NKS"-theory gives a smaller dispersion than the "NHS"-theory, the "NHS"-theory can be disregarded with good justification when making the general comparison of the theories. This means that only the theories of "N", "N and K", "N and H", and "NKS", are left. In the comparison the "NKS"-theory has displayed the least dispersion of the three theories, but this may be merely due to chance, as Table 5 shows. A fresh attempt was made to distinguish between the three theories by using the populations in Tables 3 and 4. The histograms of the results for Table 3 are shown in Figs. 21–24, and for Table 4 in Figs. 25–27. As previously, X and $\hat{\sigma}^2$ were determined for all these histograms, and the results were compared by means of a variance ratio test (see Table 6).

As is apparent from Table 6, the "N"-theory and not as before the "NKS"-theory displays the smallest dispersion. The "N"-theory was compared in the usual manner with the other six histograms. The significance levels indicate that there is a difference in dispersion between the "N"-theory and the "N and H"-theory. Furthermore, the difference between the "N"-theory and the "N and K"-theory for hardened steels might not be attributed to chance. As can be seen, the variances have increased considerably for the population according to Table 3 in comparison with the population according to Table 2. For this reason, therefore, the histograms were compared, two and two, so that the variances of the histograms in Figs. 11 and 21 were compared, as were also the variances of Figs. 17 and 22, 18 and 23, and 19 and 24. The resultant F-values were 1.38, 1.61, 1.88 and 1.74, and the corresponding significance levels, p, were 1.1, 0.01, 0.05 and 0.01 per cent. This consequently means that the test has shown that for all these three theories the larger population resulted in such a poorer agreement between β_{exp} and β_{est} that the difference is significant. Possible explanations of this may be that the three theories (which only differ from one another in

TABLE 5

Theory	N	REP	N and M	N and K	N and H	NKS	NHS
Histogram, Fig.	11	12	16	17	18	19	20
Number of bars	145	149	150	152	150	153	150
Mean, $\overline{X}$	0.920	0.937	0.851	0.836	0.855	0.834	0.849
Variance, $\hat{\sigma}^2$	0.0196	0.0261	0.0220	0.0183	0.0180	0.0161	0.0173
F	1.22	1.62	1.37	1.14	1.12	*	1.08
p	11	0.1	2.5	22	25	*	32

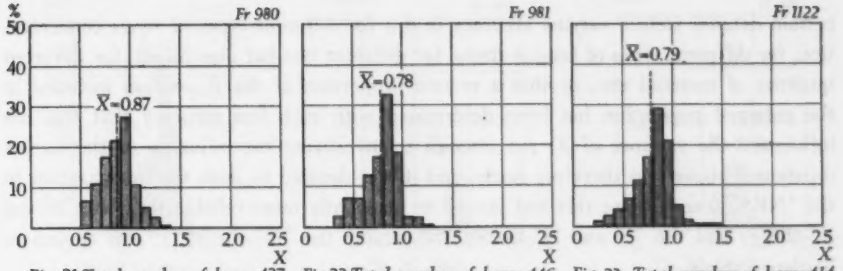


Fig. 21. Total number of bars=427. *Fig. 22. Total number of bars=446.* *Fig. 23. Total number of bars=414.*

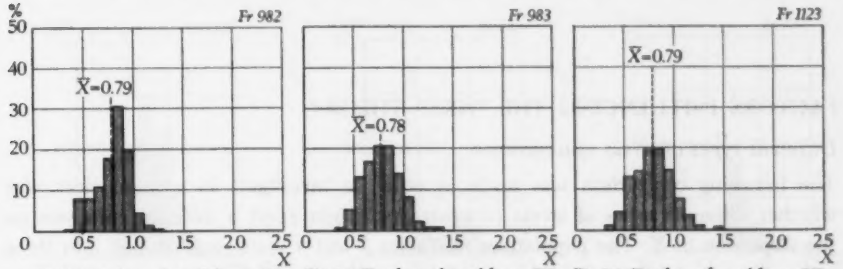


Fig. 24. Total number of bars=476. *Fig. 25. Total number of bars=510.* *Fig. 26. Total number of bars=518.*

Fig. 21. Neuber (N)
Fig. 22. Neuber–Kuhn (N and K) — Non-hardened steels
Fig. 23. Neuber–Heywood (N and H) — Non-hardened steels
Fig. 24. Modified Neuber–Kuhn (NKS) — Non-hardened steels
Fig. 25. Neuber–Kuhn (N and K) — Hardened steels
Fig. 26. Neuber–Heywood (N and H) — Hardened steels
Fig. 27. Modified Neuber–Kuhn (NKS) — Hardened steels

$\varkappa$ according to Lehr.

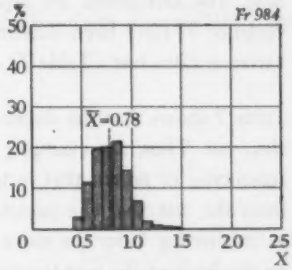


Fig. 27. Total number of bars= 493.

Figs. 21–27. Dispersion in X according to different theories.

TABLE 6

Theory	Non-hardened steels				Hardened steels		
	N	N and K	N and H	NKS	N and K	N and H	NKS
Histogram, Fig.	21	22	23	24	25	26	27
Number of bars	427	446	414	476	510	518	493
Mean, $\overline{X}$	0.866	0.777	0.792	0.791	0.775	0.786	0.784
Variance, $\hat{\sigma}^2$	0.0271	0.0296	0.0338	0.0280	0.0315	0.0367	0.0293
F	*	1.09	1.25	1.035	1.16	1.35	1.08
p	*	18	1.5	35	7	0.05	22

certain details) yield a varying accuracy in β_{est} for different types of stress concentration, for different types of fatigue stress, for different test-bar dimensions, for different qualities of material etc., or that a certain proportion of the β_{exp}-values included in the enlarged population has been determined with such less accuracy that this has influenced the variance of X. An attempt to investigate the influence of the factors mentioned above was therefore made, and it was decided to limit the investigation to the "NKS"-theory, since this had proved to be slightly more reliable than the "N and K"-theory and the "N and H"-theory and, unlike the "N"-theory, it also applies to hardened steels.

FACTORS INFLUENCING THE "NKS" THEORY

Different types of stress concentration

The following comparison was made in order to investigate in a more clear way whether different types of stress concentration might exert a differing influence on the dispersion in X. The populations in Tables 3 and 4 were each divided into three groups, namely all bars with groove, all bars with fillet and all bars with transverse hole. The histograms are reproduced in Figs. 28–33. The mean value of X and the variance $\hat{\sigma}^2$ have been determined and the histograms compared with the aid of the variance ratio test (Table 7).

Table 7 shows that the dispersion varies considerably. The variations are not, however, clear-cut. Thus, for example, the largest and the smallest dispersions occur with the same type of notch, that is to say, with test bars having a fillet. It is also apparent from the test that the population with the smallest dispersion cannot be considered as originating from the same parent distribution as the others. It is hardly possible on the basis of the test to reject one or other type of notch for having a larger dispersion in X than the others. This is scarcely to be expected, since the theoretical stress-concentration factors α are determined in the same manner for the different types of notch and since β is determined according to the same formula. A possible tendency towards greater dispersion in the hardened steels than in the non-hardened qualities may be discerned.

Different types of stress

The following comparison was made between the different types of loading in order to obtain an idea of how various types of stress influence the dispersion in the X-value. The bars of hardened and non-hardened steels, respectively, were divided into three groups, namely bars subjected to tension-compression, bending and torsion stresses. The six histograms are given in Figs. 34–39. As usual, the mean and the variance were calculated and the results are shown in Table 8.

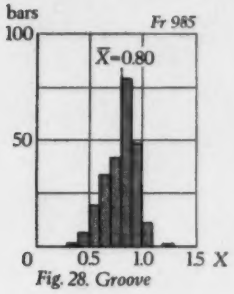


Fig. 28. Groove

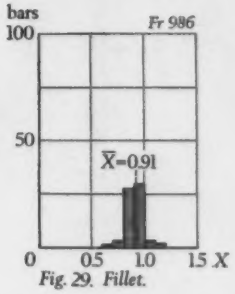


Fig. 29. Fillet.

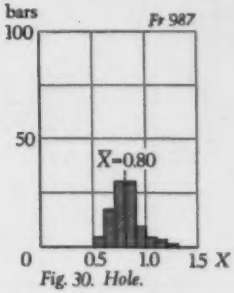


Fig. 30. Hole.

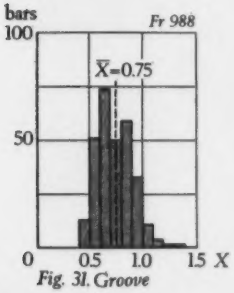


Fig. 31. Groove

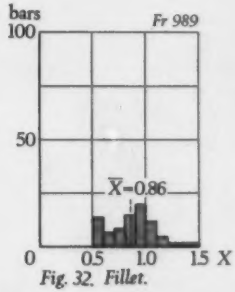


Fig. 32. Fillet.

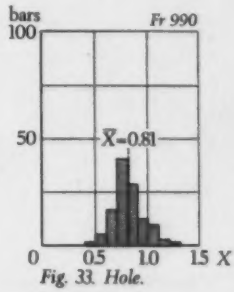


Fig. 33. Hole.

Figs. 28–33. Dispersion in X according to the modified Neuber–Kuhn theory (NKS) for different types of stress concentration.

Figs. 28–30. Non-hardened steels
Figs. 31–33. Hardened steels
$\varkappa$ according to Lehr.

TABLE 7

Material	Non-hardened steels			Hardened steels		
Theory	NKS	NKS	NKS	NKS	NKS	NKS
Stress concentration	Groove	Fillet	Hole	Groove	Fillet	Hole
Histogram, Fig.	28	29	30	31	32	33
Number of bars	239	65	98	279	78	114
Mean, $\bar{X}$	0.800	0.905	0.803	0.745	0.862	0.805
Variance, ϑ^2	0.0215	0.0103	0.0184	0.0260	0.0415	0.0197
F	2.08	•	1.78	2.57	4.03	1.9
p	0.05	•	0.7	0.05	0.05	0.25

Two variance ratio tests were performed. As can be seen from these tests, it is *not* possible to detect any tendency towards increased dispersion in X for any of the different types of loading. As usual, however, the hardened and non-hardened steels differ from one another.

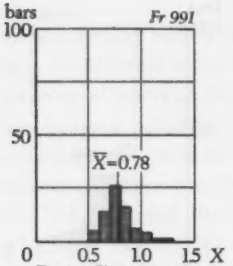


Fig. 34. Tension–compression

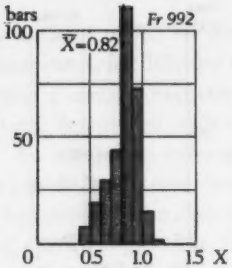


Fig. 35. Bending.

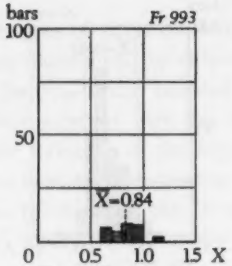


Fig. 36. Torsion.

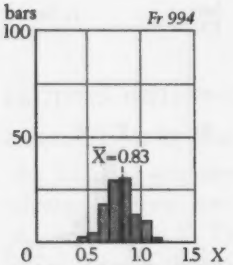


Fig. 37. Tension–compression

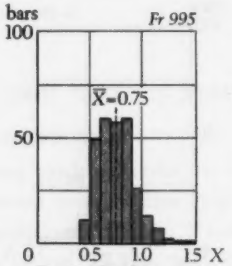


Fig. 38. Bending.

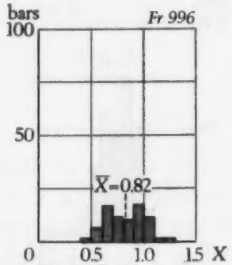


Fig. 39. Torsion.

Figs. 34–39. Dispersion in X according to the modified Neuber–Kuhn theory (NKS) for different types of stress.

Figs. 34–36. Non-hardened steels. Figs. 37–39. Hardened steels. $\varkappa$ according to Lehr.

TABLE 8

Material	Non-hardened steels			Hardened steels		
Theory	NKS	NKS	NKS	NKS	NKS	NKS
Stress	Tension	Bending	Torsion	Tension	Bending	Torsion
Histogram, Fig.	34	35	36	37	38	39
Number of bars	73	302	27	112	285	74
Mean, $\bar{X}$	0.784	0.822	0.839	0.825	0.754	0.824
Variance, $\hat{\sigma}^2$	0.0198	0.0190	0.0203	0.0221	0.0276	0.0310
F	1.04	•	1.07	1.16	1.45	1.64
p	40	•	40	17	0.1	0.2
F	—	—	—	•	1.25	1.41
p	—	—	—	•	10	5

Different test-bar dimensions

Since the absolute size according to Lehr has a considerable influence on the fatigue limit, it might be expected that the dispersion in X could vary with the dimensions of the test specimens. In order to investigate whether this was the case, the popula-

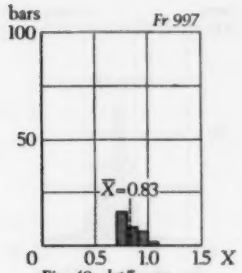


Fig. 40. $d \leq 5$ mm.

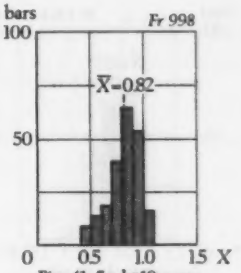


Fig. 41. $5 < d \leq 10$ mm.

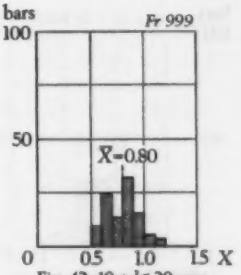


Fig. 42. $10 < d \leq 20$ mm.

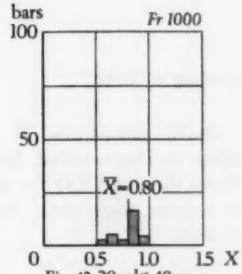


Fig. 43. $20 < d \leq 40$ mm.

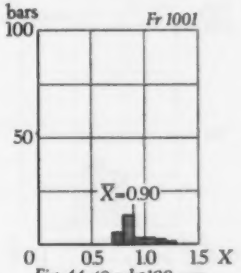


Fig. 44. $40 < d \leq 100$ mm.

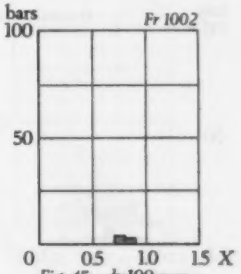


Fig. 45. $d > 100$ mm.

Figs. 40–45. Dispersion in X according to the modified Neuber-Kuhn theory (NKS) for different test-bar dimensions. Non-hardened steels.

* according to Lehr.

TABLE 9

Method	Non-hardened steels				
	NKS	NKS	NKS	NKS	NKS
Diameter, mm	$d \leq 5$	$5 < d \leq 10$	$10 < d \leq 20$	$20 < d \leq 40$	$40 < d \leq 100$
Histogram, Fig.	40	41	42	43	44
Number of bars	30	210	101	29	27
Mean, $\bar{X}$	0.827	0.820	0.797	0.802	0.902
Variance, $\hat{\sigma}^2$	0.0078	0.0214	0.0221	0.0129	0.0173
F	—	•	1.03	1.67	1.24
p	—	•	40	2	20
F	•	2.75	—	—	—
p	•	0.05	—	—	—

tions in Tables 3 and 4 were divided according to size into the following groups: $d \leq 5$ mm, $5 < d \leq 10$ mm, $10 < d \leq 20$ mm, $20 < d \leq 40$ mm, $40 < d \leq 100$ mm, and $d > 100$ mm. A histogram was formed for each group in the normal manner, and the mean and the variance were calculated as usual. The histograms are shown in Figs. 40–50. The results of the variance ratio test for certain of these histograms are to be found in Tables 9 and 10.

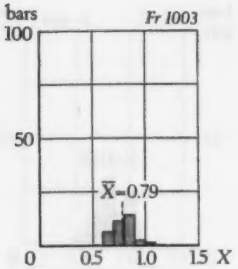


Fig. 46. $d \leq 5$ mm.

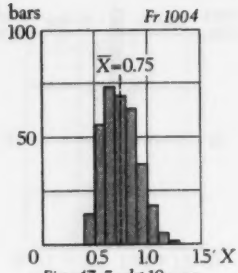


Fig. 47. $5 < d \leq 10$ mm.

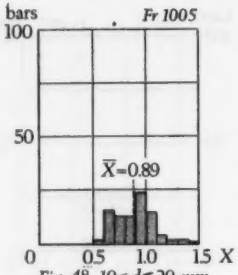


Fig. 48. $10 < d \leq 20$ mm.

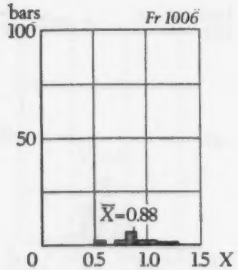


Fig. 49. $20 < d \leq 40$ mm.

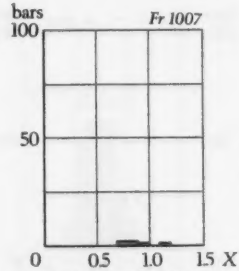


Fig. 50. $40 < d \leq 100$ mm.

$\varkappa$ according to Lehr.

Figs. 46–50. Dispersion in X according to the modified Neuber–Kuhn theory (NKS) for different test-bar dimensions. Hardened steels.

TABLE 10

Method	Hardened steels		
	NKS	NKS	NKS
Diameter, mm	$d \leq 5$	$5 < d \leq 10$	$10 < d \leq 20$
Histogram, Fig.	46	47	48
Number of bars	34	335	92
Mean, $\bar{X}$	0.794	0.747	0.892
Variance, $\hat{\sigma}^2$	0.0089	0.0268	0.0444
F	—	•	2.6
p	—	•	0.05
F	—	3.0	•
p	—	0.05	•

The results obtained with the variance ratio test must be considered as being very uncertain, since the histograms in many cases deviate to a great extent from a normal distribution. It might be possible, nevertheless, to divide the population into two groups with significantly varying dispersions in relation to one another. The limit lies at a bar diameter of about 5 mm; the variance below this diameter is significantly less than above. The variance is significantly greater for bars of hardened steel having a diameter of 10–20 mm than for bars having a diameter of 5–10 mm. In the case of non-hardened steels, however, the variance is less in the groups $20 < d \leq 100$ than in the groups $5 < d \leq 20$, and thus the increase in variance is not directly dependent on the dimensions for diameters of more than 5 mm.

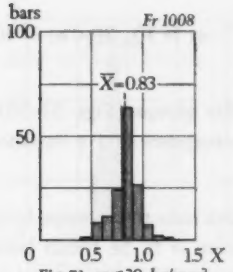


Fig. 51. $\sigma_s \leq 30$ kg/mm².

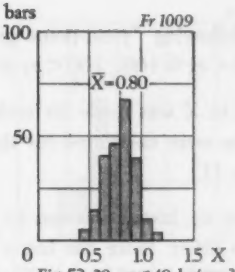


Fig. 52. $30 < \sigma_s \leq 40$ kg/mm².

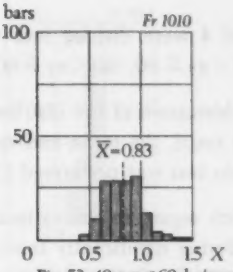


Fig. 53. $40 < \sigma_s \leq 60$ kg/mm².

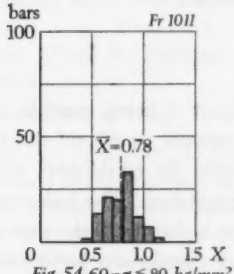


Fig. 54. $60 < \sigma_s \leq 80$ kg/mm².

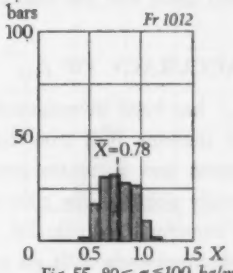


Fig. 55. $80 < \sigma_s \leq 100$ kg/mm².

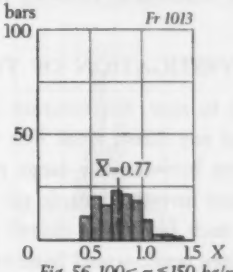


Fig. 56. $100 < \sigma_s \leq 150$ kg/mm².

Figs. 51–56. Dispersion in X according to the modified Neuber–Kuhn theory (NKS) for materials with different yield points, σ_S.

* according to Lehr.

TABLE 11

Yield point, σ_S	30	30–40	40–60	60–80	80–100	100–150
Histogram, Fig.	51	52	53	54	55	56
Number of bars	143	224	148	109	142	123
Mean, $\bar{X}$	0.833	0.796	0.829	0.783	0.780	0.765
Variance, $\hat{o}^2$	0.0168	0.0203	0.0326	0.0226	0.0227	0.0394
F	*	1.2	1.94	1.35	1.35	2.35
p	*	12	0.05	5	4	0.05

Yield point of the tested materials

When making a direct comparison between hardened and non-hardened steels (Table 6), it was not possible to detect any difference in the dispersion between the two categories. With later comparisons (Table 7 and, above all, Table 8) a significant difference has occurred, however, in the dispersion in X for the two categories of steel. The variance for hardened steels is greater than for non-hardened steels.

The following investigation was carried out to see whether this difference was in any way related to the yield point of the materials. All the test bars included in Tables 3

and 4 were divided into the following "yield-point groups": $\sigma_S \leqq 30$, $30 < \sigma_S \leqq 40$, $40 < \sigma_S \leqq 60$, $60 < \sigma_S \leqq 80$, $80 < \sigma_S \leqq 100$, $100 < \sigma_S \leqq 150$.

A histogram of the distribution in X was made for each of the groups (Figs. 51–56). As usual, the mean and variance were calculated for these histograms and a variance ratio test was performed (Table 11).

With regard to the variance, the six histograms can be divided into two groups both differing significantly from each other. Since the larger variance is to be found both at $40 < \sigma_S \leqq 60$ and $100 < \sigma_S \leqq 150$, it is not possible to observe any special trend in the relationship between the yield point and the dispersion in X.

INVESTIGATION OF THE ACCURACY OF β_{exp}

Up to now, the accuracy of β_{est} has been investigated without it being possible to find any direct weak link in the theories. It is true that occasional significant deviations between the large population and a certain group could be established, but closer investigations do not generally point to any clear-cut conclusions. The influence of each factor has indeed been examined individually, but it is hardly likely that a better result would have been obtained even with an accurate comparison performed simultaneously of all the influencing factors, which are independent of one another, in the form of a higher factorial comparison.

Hitherto the investigations have been limited to the examination of possible causes for errors in β_{est}. Since the dispersion in X is influenced to exactly the same degree by the accuracy of β_{exp} as of β_{est}, it was considered desirable first to check β_{exp} before carrying out any further work on β_{est}. As has already been mentioned, the number of specimens in the different Wöhler curves varied considerably, ranging from 2 up to about 30, and this probably had a very pronounced influence on the accuracy with which β_{exp} was determined and consequently on the dispersion in X. For the purpose of studying more closely the influence of the number of specimens per Wöhler curve, a selection was made from each of the populations in Tables 3 and 4 of the bars where the fatigue limits had been determined with more than 5 specimens per Wöhler curve. This resulted in a population of 69 bars being obtained from Table 3 and a population of 53 bars from Table 4, comprising in both cases mainly bars with groove subjected to a bending stress. The histograms for the new populations are shown in Figs. 57 and 58. If these are compared with the histograms for Tables 3 and 4, Figs. 24 and 27, the greatly decreased dispersion is immediately apparent. The result of the usual variance ratio test is shown in Table 12. For the sake of comparison, this Table also includes the original population from Table 1 (Fig. 19).

The dispersion for non-hardened steels decreases to such an extent that the deviations from both histograms in Figs. 19 and 24 are of significance, with the possible exception of the histogram in Fig. 19. The dispersion for hardened steels, on the other hand, has not decreased sufficiently to be of any significance. This result gives grounds

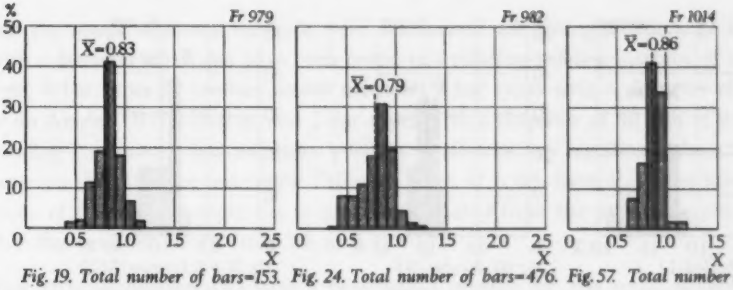


Fig. 19. Total number of bars=153. *Fig. 24. Total number of bars=476.* *Fig. 57. Total number of bars=69.*

Figs. 19, 24, 57. Non-hardened steels

Figs. 27 and 58. Hardened steels

$\varkappa$ according to Lehr.

Figs. 19, 24, 57, 27, 58. Comparison of the dispersion in X for populations with varying accuracy in the experimentally determined notch factor, β_{exp}.

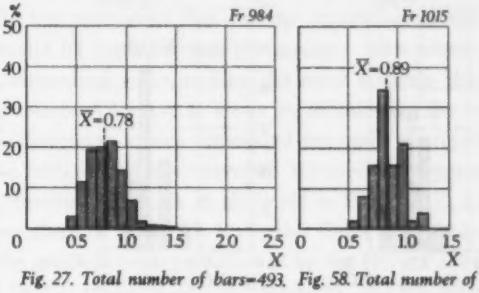


Fig. 27. Total number of bars=493. *Fig. 58. Total number of bars=53.*

TABLE 12

Material	Non-hardened steels			Hardened steels	
Method	NKS	NKS	NKS	NKS	NKS
Histogram, Fig.	19	24	57	27	58
Number of bars	153	476	69	493	53
Mean, $\overline{X}$	0.834	0.791	0.860	0.784	0.892
Variance, $\hat{\sigma}^2$	0.0161	0.0280	0.0097	0.0293	0.0221
F	1.66	2.9	*	3.02	2.28
p	1	0.05	*	0.05	0.05
F	—	—	—	1.32	*
p	—	—	—	10	*

for believing that the number of specimens per Wöhler curve, that is to say, the accuracy with which β_{exp} is determined, is of decisive importance for the dispersion in X, in other words, a decisive factor when checking the accuracy of the different theories for determining β_{est}. For this reason, therefore, a larger test series was commenced at the Asea laboratories in Västerås and Stockholm, partly with the intention of making a careful check of different β_{est}-values with the aid of accurately determined fatigue strength for the more usual standard steels. Between 25 and 30 specimens were selected for each Wöhler curve. At present, 14 β_{exp}-values are available. In addition, a further 4 β_{exp}-values with more than 20 specimens per Wöhler curve have been traced in the literature. As usual, a histogram was made for this population of 18 bars,

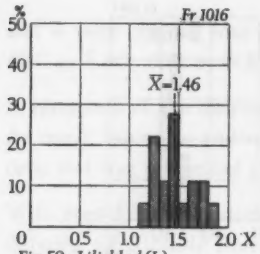


Fig. 59. Liljeblad (L).

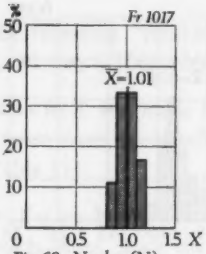


Fig 60. Neuber (N).

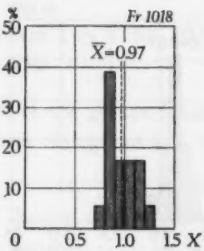


Fig. 61. R.E. Peterson (REP).

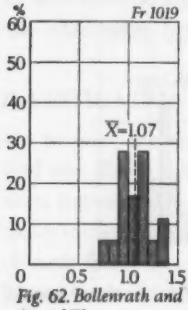


Fig. 62. Bollenrath and Troost (B and T).

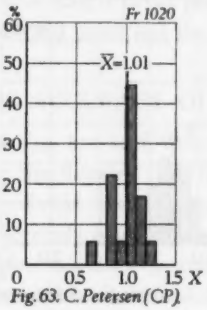


Fig. 63. C. Petersen (CP).

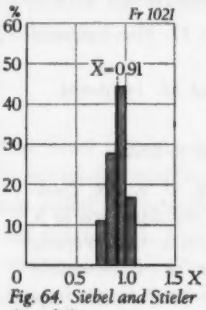


Fig. 64. Siebel and Stieler (S and S).

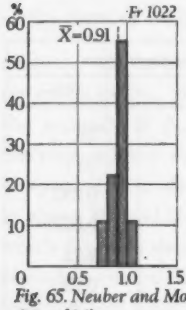


Fig. 65. Neuber and Moore (N and M).

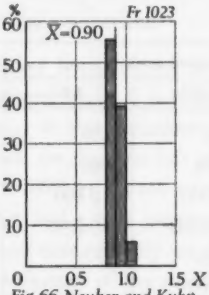


Fig. 66. Neuber and Kuhn (N and K).

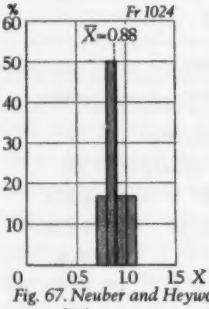


Fig. 67. Neuber and Heywood (N and H).

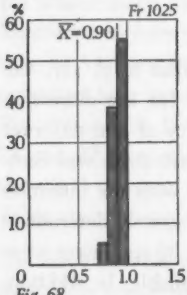


Fig. 68. Modified Neuber-Kuhn (NKS).

x according to Lehr, in Fig. 63 according to C. Petersen.

Figs. 59–68. Comparison of the dispersion in X based on tests with higher accuracy in the experimentally determined notch factor, β_{exp}.

and the mean value and variance were determined. Of this population, 11 bar types with a diameter of 10 mm have been tested in a rotating-bending machine of cantilever type, 4 bar types in another similar machine, 2 bar types with a diameter of 7.5 mm in an Amsler UHF pulsator, and 2 bar types with a diameter of 40 mm in a rotating-bending machine. It has not been possible to observe any significant deviation in X between the different test series. Different types of notch have also been investigated. None of these notch types has in any way deviated from the others, despite the fact that the α-values varied from 1.3 to 5.1.

The variance decreased in such a pronounced manner for this small population that it was found to be significantly less than even that for the population of the histogram in Fig. 7. From this it should be apparent that the accuracy with which β_{exp} is determined is of the greatest importance when seeking the most accurate theory for determining β_{est}, that is, if the selection is made as above by establishing the variance $X = \beta_{exp}/\beta_{est}$. For this reason, therefore, it was considered necessary to repeat once again the complete comparison between all the described theories. Histograms were made as usual for each of the theories. These are to be found in Figs. 59–68. Data for the usual variance ratio test are given in Table 13. From this Table it is evident that the "NKS" theory still yields the smallest variance, followed by the "N and K" and "N and H" theories. Data for the "NHS" theory were also calculated, and the mean value $\overline{X} = 0.882$ and the variance $\hat{\sigma}^2 = 0.0047$ were obtained. The variance is thus larger than for the "N and K" theory, and consequently the "NHS" theory has not been included in the Table. The theories according to "L", "REP", "B and T", and "CP" can be eliminated. All the theories based on Neuber and, in addition, the "S and S" theory have such small differences with regard to $\hat{\sigma}^2$ that the variations may possibly be due to chance. Since the "NKS" theory, however, with only one exception, has displayed the smallest dispersion for nearly all the comparisons, it can be considered most likely that this theory will yield the most reliable agreement between β_{est} and β_{exp} and can thus be accepted as being suitable for calculating notch factors when a machine part is to be dimensioned for fatigue.

The results from Table 13 diminish the values of the comparisons in Tables 7–11, since these contain β_{exp}-values which have not been determined with satisfactory

TABLE 13

Material	Non-hardened steels									
Method	L	N	REP	B and T	CP	S and S	N and M	N and K	N and H	NKS
Histogram, Fig.	59	60	61	62	63	64	65	66	67	68
Number of bars	18	17	18	18	18	18	18	18	18	18
Mean, $\overline{X}$	1.462	1.005	0.972	1.066	1.006	0.906	0.910	0.901	0.882	0.898
Variance, $\hat{\sigma}^2$	0.0345	0.0072	0.0194	0.0230	0.0197	0.0077	0.0071	0.0039	0.0062	0.0036
F	9.7	2.0	5.4	6.4	5.5	2.2	2.0	1.1	1.74	*
p	< 0.05	8	0.06	< 0.05	0.05	5.5	8	40	13	*

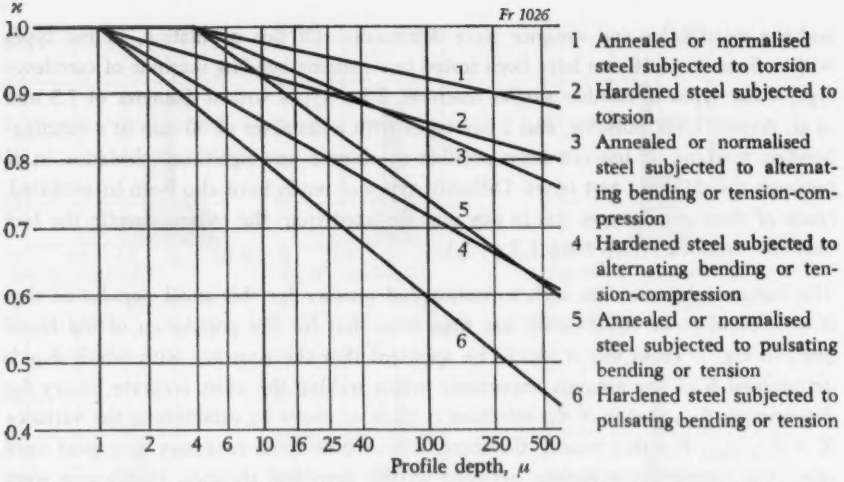


Fig. 69. The surface-finish fatigue factor, $\varkappa$, according to Gaier, as a function of the profile depth.

accuracy. Until a sufficient number of accurate β_{exp}-values have been determined or until the nature of fatigue has become sufficiently well known, the comparisons made can give nevertheless a certain idea of how the accuracy of the selected theory ("NKS") is to be judged.

INFLUENCE OF THE PROFILE DEPTH ON THE FATIGUE STRENGTH

Up to now, the same profile depth factor $\varkappa$ has been used in all the histograms, namely that proposed by Lehr. In a recently published dissertation, Gaier [62] has put forward, however, a new method of taking into account the influence of the profile depth on the fatigue strength. Fig. 69 contains a family of curves for $\varkappa$ according to Gaier. The populations in Figs. 57 and 58 were chosen to investigate which of these methods is the most reliable. A selection was made of the bars from the two populations for which Lehr and Gaier give different values of $\varkappa$. In this way, 21 bars were obtained from each of the histograms in Figs. 57 and 58, and one histogram was made for each of these with $\varkappa$ according to Lehr and one with $\varkappa$ according to Gaier. These are shown in Figs. 70, 71, 72 and 73. The result of the variance ratio test is given in Table 14.

A very small improvement was obtained in both cases with $\varkappa$ according to Gaier, but this is very probably due to chance. Since Gaier's $\varkappa$-values are slightly simpler to use, his theory may be considered preferable.

INFLUENCE OF THE GROOVE ANGLE ON THE DISPERSION IN X

Table 3 includes in all 286 grooved bars. Of these bars 145 have a groove angle of $\omega \neq 0°$. In order to check the influence of Neuber's factor $\pi/(\pi - \omega)$ on the disper-

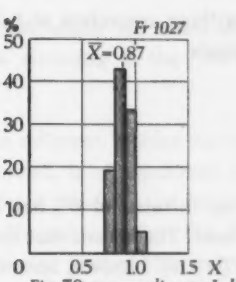


Fig. 70. ϰ according to Lehr. Total number of bars= 21.

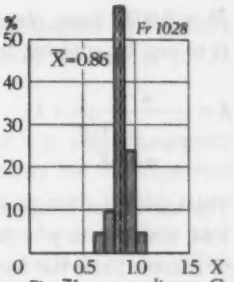


Fig. 71. ϰ according to Gaier. Total number of bars= 21.

Figs. 70 and 71. Non-hardened steels

Figs. 72 and 73. Hardened steels

β_{est} according to NKS. More than five specimens per Wöhler curve.

Figs. 70–73. Comparison between different surface-finish fatigue factors, ϰ.

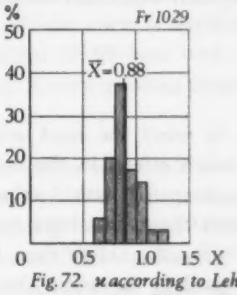


Fig. 72. ϰ according to Lehr. Total number of bars= 35.

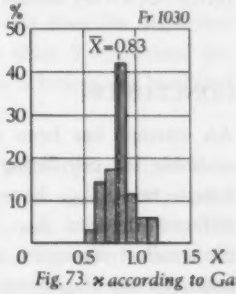


Fig. 73. ϰ according to Gaier. Total number of bars= 35.

TABLE 14

Material	Non-hardened steels		Hardened steels	
Method	NKS *Lehr*	NKS *Gaier*	NKS *Lehr*	NKS *Gaier*
Histogram, Fig.	70	71	72	73
Number of bars	21	21	21	21
Mean, $\overline{X}$	0.87	0.86	0.88	0.83
Variance, $\hat{\sigma}^2$	0.0073	0.0069	0.0184	0.0174
F	1.06	•	1.06	•
p	45	•	45	•

sion in X, X-values of these 145 bars were recalculated with this factor, and the two populations were compared with each other. The population without $\pi/(\pi-\omega)$ gives a slightly larger variance, $\hat{\sigma}^2 = 0.0405$ in comparison with 0.0374 for the population with the factor $\pi/(\pi-\omega)$.

The variance ratio becomes 1.08 and the significance level p becomes about 30 per cent, that is to say the improvement resulting from $\pi/(\pi-\omega)$ is very insignificant.

In order to investigate the influence of the groove-angle correction on the dispersion in X for the "NKS" theory, that part of the population from the histogram in Fig. 68 where $\omega \neq 0$ was studied. The population comprised 8 bar types. This resulted in

$\hat{\sigma}^2 = 0.0032$ being obtained without correction and $\hat{\sigma}^2 = 0.0046$ with correction, that is to say, by applying the formula

$$\beta = \frac{\alpha - 1}{1 + \frac{\pi}{\pi - \omega}\sqrt{\frac{\varrho^*}{\varrho}}} + 1$$

The correction brings about an increase of $\hat{\sigma}^2$, but the variance ratio becomes only 1.44, which gives $p = 32$ per cent. This means that the deterioration can be attributed to chance. Since the factor $\pi/(\pi - \omega)$ involves, however, a more complicated formula, it is proposed that it should be rejected. Kuhn and Hardrath suggest too that the factor $\pi/(\pi - \omega)$ should be rejected when determining β [10].

CONCLUSION

An attempt has been made to select the most accurate of the different theories available for calculating the notch effect. In the course of this, the β-values from fatigue tests, β_{exp}, have been compared with β-values determined according to the different theories, β_{est}. The best theory has been considered to be the one yielding the smallest dispersion in $X = \beta_{exp}/\beta_{est}$. More than 1000 experimentally determined notch factors β_{exp} have been used in the tests. The great majority of these values have been obtained from the available literature, but a number of β_{exp}-values have also been determined at the Asea laboratories in Västerås and Stockholm. In this latter case, the number of bars per Wöhler curve amounted to between 20 and 30, and consequently rather accurate β_{exp}-values were obtained. With four exceptions, it has not been possible to achieve this high accuracy with the large population of 1000 β_{exp}-values. In order to avoid calculating 1000 β_{est}-values for each of the investigated theories, a series of comparative tests (Tables 2 and 5) was carried out on arbitrarily selected populations, after which a more thorough test was made on those theories which gave evidence of the best results during the first experiments. (Tables 6, 12 and 13.) The following conclusions can be drawn from the comparisons.

1. The theories based on Neuber's formula for determining β displayed the smallest dispersion in X, that is to say, the best agreement between β_{exp} and β_{est}. The best of these theories based on Neuber proved to be the one where the material constant ϱ^* has been formulated as a function of the yield point σ_S. This theory ("NKS") differed in a significant manner from some of the other theories. The departure from the other theories could be attributed to randomness. Since the "NKS" theory has given evidence of the smallest dispersion, it can be considered, nevertheless, to be the most suitable one for application when a part is to be dimensioned for fatigue in the presence of notches.

2. The dispersion in X for the "NKS" theory is significantly larger for hardened than for non-hardened steels, which gives reason to believe that the hardened steels are

affected by other factors than those which are taken into consideration by Neuber's formula for β with modifications according to the "NKS" theory. (Tables 6, 10, 12 and 14.)

3. The number of test bars in the different Wöhler curves, that is to say, the accuracy with which β_{exp} has been determined, is of significant importance for the dispersion in X. Thus, the dispersion in X for a population of 69 bars, where each Wöhler curve contains more than 5 bars, is significantly less than the dispersion for populations containing 153 and 476 bars, respectively, without this restriction (Table 12). The difference between the corresponding populations for hardened steels is hardly significant. The dispersion in X for non-hardened steels for a population containing 18 bars with more than 20 specimens in each Wöhler curve is significantly less than the dispersion for the above-mentioned population of 69 bars with more than 5 specimens per Wöhler curve. (Table 13, Fig. 68.) A corresponding comparison is lacking for hardened steels.

4. In the preceding paragraph mention was made of two "NKS" populations containing 153 and 476 bars, respectively, for which the dispersion was significantly larger than for a population with accurately determined Wöhler curves. It turns out, furthermore, that these two populations differ significantly without any restrictions from each other with regard to the dispersion in X. Since the smaller population has been arbitrarily selected from the larger one, attempts were made to find the possible explanations for the significant difference in the dispersion. These included investigations to see whether the dispersion varies for different types of stress concentration or stress, for different dimensions of test bar, or whether the dispersion varies with increasing values of the yield point of the tested material. No significant results, however, were obtained (Tables 7–11).

5. The accuracy of the theoretical stress-concentration factor, α, naturally influences also the dispersion in X. The reason for the difference in the dispersion between the two populations investigated in paragraph 3 might also be related to errors in α. After making a more thorough examination, it became apparent also that the α-values for a considerable proportion of the population containing 476 bars were greater than 4, and that these to a large extent were to be found in the interval X less than 0.6. The increased dispersion in X ought thus to be due to the fact that the α-values greater than 4 are too high so that the corresponding β-values are also too high, resulting in too low X-values. Against this assumption, however, it can be said that in the population of 18 bars where more than 20 specimens are included in the Wöhler curves there are three α-values of 4.7, 4.7 and 5.1, respectively, and the corresponding X-values are in the middle of the population.

6. When calculating β, it does not appear necessary to take into consideration whether the angle in a notch deviates from 0°, which Neuber states should be done. This has already been pointed out by Kuhn.

7. The influence of the profile depth can be compensated by adopting a factor $\varkappa$ for which both Lehr and Gaier have given curves (Table 14). Both these curves are similar; it is true that Gaier's curve displays a slightly smaller dispersion in X, but the difference is not significant. Gaier's curve may be preferred, since his $\varkappa$-values are slightly simpler to apply.

APPLICATION

The International Institute of Welding has made a number of comments in a recently published document about the safety factor and efficiency of welded joints subjected to static forces [79]. Most of the concepts in this document can be applied also, with suitable modifications, to machine parts subjected to fatigue stresses. For this purpose, the strength, C, of the part is calculated. Such a calculation is based on, for example, the fatigue strength of the material used, σ_D, the stress-concentration factor, α, determined with regard to the shape of the part, and from this also the fatigue-notch factor, β, the influence of the surface-finish fatigue factor, $\varkappa$, and the influence of the dimensional fatigue factor, λ.

Since all these values, σ_D, β, $\varkappa$ and λ, are based on empirical results, they are naturally not exact, but consist, in principle, of a mean value and a dispersion around this value. Information about these different values must be obtained from various sources. Thus, fatigue tests will provide information about the mean value of the fatigue strength σ_D and the dispersion of the individual values around this mean value. The accuracy of the factors β, $\varkappa$ and λ can be obtained from this paper. The accuracy of λ has not been checked with the same precision as for β and $\varkappa$, since the necessary data for this are lacking. If β is determined according to the "NKS" theory, and $\varkappa$ and λ according to Figs. 6 and 7, respectively, the total effect of these factors will be on the average slightly too high, namely $1/0.898 = 1.11$ times (Table 13, column "NKS"). The dispersion of these factors together can be obtained from the same Table, namely $\sqrt{0.0036} = 0.06$. These two values apply to carbon steels. For hardened steels a dispersion of 0.09 and a correction factor of $1/0.898 = 1.12$ (Table 12) would probably be more correct (Figs. 57 and 58). These conclusions assume that the results of the investigations are representative also for a large population.

The factors influencing C can be combined to form a mean value of the strength ($\bar{C}$) and dispersion ($\hat{\sigma}_C$) around this value (Fig. 74, where for the sake of simplicity a normal distribution has been assumed).

In the same way, the "load", L, is assumed to comprise a similar normal distribution ($\bar{L}$, $\hat{\sigma}_L$, see Fig. 74). It would then be possible to use the four quantities $\bar{C}$, $\hat{\sigma}_C$, $\bar{L}$ and $\hat{\sigma}_L$ for determining a suitable relationship between C and L. Assuming normal distributions, the governing factor would be given by

$$\frac{\bar{C}-\bar{L}}{\sqrt{\hat{\sigma}_C^2+\hat{\sigma}_L^2}}=\frac{\bar{C}-\bar{L}}{\hat{\sigma}}$$

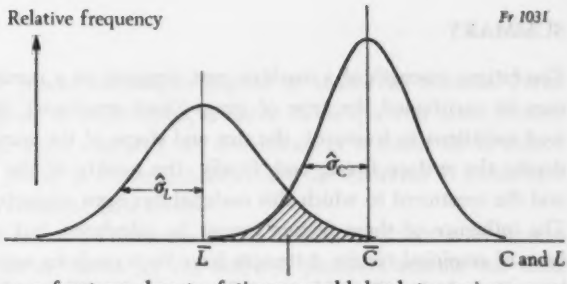


Fig. 74. Relationship between strength, C, load, L, and fracture risk.

In the case of a part where a fracture due to fatigue would lead to serious consequences, the expression $\bar{C} - \bar{L}$ is selected so that a sufficiently low fracture risk is obtained. On the other hand, in the case of a component where a certain percentage of fatigue failures can be tolerated, a lower value of $\bar{C} - \bar{L}$ is selected. An optimum dimensioning can thus be obtained with regard to both price and suitability.

According to conventional tables of normal distributions, a value of $(\bar{C} - \bar{L})/\hat{\sigma} = 1.28$, for example, will give a fracture risk of 10^{-1}, whereas a corresponding value of 4.75 gives the fracture risk 10^{-6}.

The application of the above-mentioned principles presents, in practice, certain difficulties, among which one of the most important is that the load very seldom has a constant amplitude, mean value and frequency. On the contrary, machine parts are generally subjected to repeated stresses of varying amplitude under a mean stress level, which is also varying. Furthermore, certain parts are subjected to occasional stresses over and above the fatigue strength. The endurance strength under such conditions will differ from that obtained at constant stress amplitude.

If machine parts subjected to such fatigue are to be correctly dimensioned, it is essential both that the problem of cumulative damage is solved and that information is obtained concerning the load spectrum for different types of dynamically loaded design.

Much research is being devoted to the problem of cumulative damage [80]. Load spectra are also being investigated in an increasing number of fields. Comprehensive results are already available from investigations performed on aircraft and vehicles [81]. Consequently, information about $\bar{L}$ and $\hat{\sigma}_L$ is accumulating now within these fields, and similar work on cranes is being prepared by, for instance, the International Institute of Welding.

Provided that these investigations lead to practical results, it will be possible, therefore, to dimension a structure or machine part according to the principles indicated in this paper. The advantage of the new principles is that it will be possible to obtain an idea of how the individual factors, including their dispersions, influence the dimensioning. Such knowledge enables the above-mentioned factor $(\bar{C} - \bar{L})/\hat{\sigma}$ to be determined according to statistical rules. The result will be that the structure or machine part will be adequately dimensioned both from the economic and the strength points of view.

SUMMARY

The fatigue strength of a machine part depends on a number of factors. Among these may be mentioned the type of stress (load spectrum), the desired life (number of load variations to fracture), the size and shape of the section under stress, the profile depth, the surface finish, and, finally, the quality of the material used for the part and the treatment to which this material has been subjected.

The influence of these factors cannot be calculated, but must be determined on the basis of empirical results. Attempts have been made by applying such results to formulate rules which will enable the influence of the different factors to be taken into consideration. Interest has been mainly concentrated to the dependence of fatigue strength on stress concentrations (notch effect).

The present paper gives an account of an attempt made to compare various formulae for calculating the notch effect resulting from stress concentrations in the form of notches, fillets and holes. In order to make this comparison impartial, it has been performed with the aid of statistics. Since the data forming the basis of the theories and the investigation are empirical, the result cannot naturally be exact. The testing machines yield certain errors, and this must be considered. The accuracy of each of the theories is given in the form of a mean value and a dispersion. When evaluating the theories, preference is given to that one having the smallest dispersion.

The paper shows that the best theory has been formulated by Neuber. With certain modifications suggested by Kuhn and Hardrath, and others, this theory yields such a good accuracy that the uncertainty expressed as a best estimate of the dispersion of a calculated value of the notch factor will be about 6 per cent for normal carbon steels and about 9 per cent for hardened steels.

On the basis of this result, a method of calculation developed with statistical reasoning is described. Provided that both the load and the strength of a part can be expressed each with its mean value and its dispersion, it is possible to select the most economical safety factor (ratio between the strength and the load). A low safety factor results in, for example, small dimensions at a low price, but a high risk of fracture. When making such calculations, it is assumed that the dispersion in the fatigue strength, the accuracy when determining the notch factor, the load spectrum of the machine part being designed, etc., are known.

The importance of all the investigations being performed with sufficient care became clearly apparent during their evaluation. Thus, for example, the Wöhler curves on which the empirical data are based must originate from a sufficiently large number of test specimens. A significant difference was established between results from, on the one hand, test series having a small number of specimens per Wöhler curve (> 5) and, on the other hand, from those test series having more than 20 specimens per Wöhler curve.

RÉSUMÉ

La résistance à la fatigue de pièces mécaniques dépend de nombreux facteurs. Parmi ces facteurs, il faut citer le genre de sollicitation, la durée de vie que l'on désire obtenir (nombre de cycles de charge avant rupture), les dimensions et la forme de la section sollicitée, la profondeur du profil, le traitement superficiel du matériau et finalement le genre de matériau et le traitement qu'il a subi.

Il est impossible de fixer par le calcul l'influence de ces facteurs et il faut avoir recours à des données empiriques. En se basant sur ces données, plusieurs auteurs ont tenté d'établir des règles de calcul qui permettent de tenir compte des différents facteurs. L'intérêt s'est surtout porté sur l'influence de concentrations existantes de tensions (effet d'entaille) sur la résistance à la fatigue.

La présente étude a pour objet de comparer quelques formules de calcul de l'action locale qui résulte de concentrations de tensions au droit de gorges, d'épaulements et de trous. Pour que la comparaison soit impartiale, elle a été effectuée par des moyens statistiques. Comme la base des théories et des recherches est empirique, le résultat ne peut évidemment être exact. Entre autres, interviennent les imperfections des machines d'essai utilisées. La précision de chacune des théories est caractérisée par une valeur moyenne et une dispersion, et on accorde la préférence à la théorie qui donne la dispersion la plus réduite.

On constate que la meilleure théorie est celle qui a été établie par Neuber. Avec certains compléments, notamment de Kuhn et Hardrath, cette théorie donne une précision telle que l'insécurité exprimée sous forme de la meilleure estimation de la dispersion d'une valeur calculée du facteur de localisation est d'environ 6 % pour l'acier au carbone ordinaire et d'environ 9 % pour l'acier trempé.

En partant de ce résultat, l'auteur esquisse une méthode de calcul basée sur un raisonnement statistique. En supposant que la charge ainsi que la résistance d'une pièce mécanique puissent s'exprimer chacune par une valeur moyenne et une dispersion, on peut choisir le coefficient de sécurité (rapport entre la résistance et la charge) de la façon la plus économique. Un faible coefficient de sécurité donne p. ex. des dimensions réduites et peu coûteuses mais un grand risque de rupture. Le calcul suppose notamment que l'on connaît la dispersion de la résistance (résistance à la fatigue), la précision de calcul du facteur de localisation et le genre de sollicitation de la pièce calculée. L'étude montre clairement qu'il importe que toutes les recherches soient effectuées avec un soin suffisant. C'est ainsi p. ex. que les courbes de Wöhler sur lesquelles sont basées les données empiriques doivent être relevées sur un nombre suffisant d'éprouvettes. Une différence importante apparaît entre le résultat obtenu, d'une part avec un faible nombre d'éprouvettes (< 5) par courbe de Wöhler et d'autre part, avec plus de 20 éprouvettes par courbe de Wöhler.

ZUSAMMENFASSUNG

Die Dauerfestigkeit einer Konstruktion hängt von einer Anzahl verschiedener Faktoren ab. Hierzu gehören Belastungsart (Lastspiel) und geforderte Lebensdauer (Anzahl der Lastwechsel bis zum Bruch), Größe und Form des belasteten Querschnittes, Rauhtiefe, Oberflächenbehandlung des Materials und schließlich Art des Werkstoffes und seine Vorbehandlung.

Der Einfluß dieser Faktoren kann nicht errechnet, sondern nur empirisch ermittelt werden. Unter Zugrundelegung bereits vorhandener Ergebnisse wurde eine Reihe von Untersuchungen vorgenommen, um Berechnungsformeln aufstellen zu können, die den Einfluß der verschiedenen Faktoren berücksichtigen. Das Hauptinteresse galt hierbei der Abhängigkeit der Dauerfestigkeit von auftretenden Spannungskonzentrationen (Kerbwirkung).

Der vorliegende Aufsatz beschreibt einen Versuch zum Vergleich verschiedener Formeln für die Berechnung der Kerbwirkung, die durch Spannungskonzentrationen infolge von Einschnitten, Absätzen und Bohrungen auftritt. Um den Vergleich objektiv zu gestalten, wurde er mit statistischen Mitteln durchgeführt. Da die Theorien und die Untersuchung selbst auf empirischen Werten fußen, kann das Versuchsergebnis natürlich nicht exakt werden. Fehlerquellen treten u. a. in den verwendeten Prüfmaschinen auf. Die Genauigkeit jeder einzelnen Theorie wird deshalb durch Mittelwert und Streuung bestimmt, und bei der Auswertung wird der Theorie mit der geringsten Streuung der Vorzug gegeben.

Es zeigt sich, daß die beste Theorie von Neuber aufgestellt worden ist. Mit bestimmten Erweiterungen u. a. von Kuhn und Hardrath ergibt diese Theorie eine so gute Genauigkeit, daß die Unsicherheit, ausgedrückt als bester Schätzwert der Streuung für einen errechneten Wert des Kerbfaktors, für normale Kohlenstoffstähle etwa 6 % und für gehärtete Stähle etwa 9 % beträgt.

Von diesen Ergebnissen ausgehend wird eine statistische Berechnungsmethode angegeben. Unter der Voraussetzung, daß sich sowohl Belastung als auch Festigkeit einer Konstruktion je durch ihren Mittelwert und ihre Streuung ausdrücken lassen, kann der Sicherheitsfaktor (Verhältnis zwischen Festigkeit und Belastung) nach wirtschaftlichen Gesichtspunkten gewählt werden. Ein niedriger Sicherheitsfaktor bedeutet z. B. geringe Abmessungen und damit niedrigen Preis, jedoch erhöhte Bruchgefahr. Die Berechnung setzt u. a. voraus, daß die Streuung der Dauerfestigkeit, die Genauigkeit bei der Bestimmung des Kerbfaktors und das Lastspiel für die berechnete Konstruktion bekannt sind.

Aus der Auswertung ging einwandfrei hervor, wie wichtig es ist, alle Untersuchungen mit genügender Sorgfalt durchzuführen. Die Wöhlerkurven z. B., auf denen die empirischen Werte beruhen, müssen mit einer ausreichenden Anzahl von Probestäben aufgenommen werden. Zwischen den Ergebnissen von Versuchen mit einerseits einer geringen Zahl (< 5) von Probestäben je Wöhlerkurve und andererseits mehr als 20 Stäben je Wöhlerkurve trat ein signifikativer Unterschied auf.

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